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REDSHIFT AND THE SHAPE OF THE UNIVERSE

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Abstract

Hubble's observation in 1929 that redshifts of far-away objects increase with their distance is customarily interpreted as being due to expansion of the universe, leading to the universally accepted ideas of the Big Bang and a spatially flat, infinite universe. We explore an alternative model of the universe, proposed by Segal in 1972, which has geometry $\mathbb{R} \times \mathrm{S}^3$. It is eternal, not expanding, and is spatially curved, compact and finite, as in the Einstein static universe. Our preliminary analysis of open source datasets shows that the model's predictions are consistent with two important types of cosmological data: cosmological redshift and cosmic background radiation. With new data from the James Webb Space Telescope, verification of predictions that distinguish the standard model from Segal's model of the universe is increasingly feasible.

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INTRODUCTION

In 1929, Hubble [1] showed that the redshift in the light spectra of far-away nebulae, first observed by Slipher, is proportional to their distances from Earth. Hubble suggested that in the de Sitter cosmology this effect could be analogous to the familiar Doppler effect, except this cosmological redshift is not caused by objects receding from each other in space, but by the stretching of space itself. In 1935, Hubble and Tolman [2] considered the expansion of the universe as one of possible explanations for the redshift-distance relation observed by Hubble. They also mentioned a possibility that the increase of redshift with distance could be caused by some other unknown effect due to the geometry of the universe. However, no definitive explanation of this kind was found at the time, and the expanding universe hypothesis became accepted as a fact.

In the Lambda Cold Dark Matter Model (ΛCDM), also known as the Standard Cosmological Model (SCM), acceptance of this hypothesis led to the ideas of the Big Bang and inflation. The shape of the universe is assumed to be spatially flat, $M_0 = \mathbb{R} \times \mathbb{R}^3$. In 1972, Irving Segal proposed an alternative explanation for the observed increase of redshifts with the distance of far-away objects [3]. The axioms of physical symmetries—global isotropy and homogeneity of space and time, and its causality properties, are satisfied not only by Minkowski spacetime, $\mathbb{R} \times \mathbb{R}^3$, but also by a universe whose geometry is $\mathbb{R} \times \mathrm{S}^3$. It is the geometry of the Einstein static universe - non-expanding, spatially closed, finite, and eternal. Einstein abandoned this model after the increase of redshift with distance became accepted as evidence for expansion of the universe. The two geometries are indistinguishable locally, even across intergalactic distances, with observable differences appearing only on cosmological scales.

The two universes and their causal structure are deeply connected, and their relationship gives rise to possible observable differences. Not only is $\mathbb{R} \times \mathbb{R}^3$ the tangent space to each observer in $\mathbb{R} \times \mathrm{S}^3$, but Minkowski space can be causally embedded in Segal's universe. This means that an observer of events in $\mathbb{R} \times \mathrm{S}^3$ would observe a time-orientation preserving set of events by observing their stereographic projection onto their local Minkowski space, tangent to $\mathbb{R} \times \mathrm{S}^3$ at observer's location. Segal postulates that observations are indeed made in this local Minkowski projection, and from this hypothesis, shows that redshift arises naturally. This theory provides a verifiable prediction for the dependence of this geometric redshift on the geodesic distance light travels through in $\mathbb{R} \times \mathrm{S}^3$, which is distinct from the redshift-distance relation provided by the ΛCDM theory.

A concise introductory overview of Segal's theory was given by Daigneault and Sangalli [4]. A detailed review of Segal's book was written by Taub [5]. References to Segal's papers and books on Chronometric Cosmology can be found in the bibliography[3,6-34].

Segal's work was not accepted by his contemporaries. They raised both theoretical and empirical concerns about chronometric cosmology [16, 25], which Segal addressed [17, 26], but the conversation died out. In the modern day, with newly available data that is more precise and farther reaching, we seek to reopen the question of chronometric cosmology and consider if it can be falsified in the modern context. Surprisingly, the currently available data does not falsify Segal's model.

SEGAL'S CHRONOMETRIC COSMOLOGY

Segal's original motivation was to explore possible generalizations of Minkowski space-time of special relativity to some other 4-dimensional manifold, given that Maxwell's equations are not only Lorentz invariant but also conformally invariant. The Poincaré group and Minkowski space-time would then be a limiting case of a more accurate theory, similarly to the Galilean group being a limit of the Lorentz group when the speed of light approaches infinity.

Lie algebras of pseudo-orthogonal groups $\mathrm{O}(1,5), \mathrm{O}(2,4)$, and $\mathrm{O}(3,3)$ are deformable into that of the fundamental dynamical variables (momenta, boosts, and space-time coordinates) in relativistic quantum mechanics [6] . As pointed out by Segal [7], $\mathrm{O}(1,5)$ which is the group of de Sitter space, is difficult to reconcile with the principle of positivity of the energy in quantum mechanics as it does not have a self-adjoint generator correspond to a nonnegative energy in any nontrivial unitary representation of $\mathrm{O}(1,5)$. The group $\mathrm{O}(2,4)$, the conformal group of Minkowski space is a candidate for a more accurate higher symmetry group as it is free from this deficiency. In 1971, Segal observed that the acausality of conformal spacetime could be remedied through its replacement by the locally identical section of the universal covering space [8].

Segal's cosmology [13] is based on the following assumptions:

- space-time is 4-dimensional manifold
- space-time has causal structure: (a) at each point of the universe there is given a convex cone of infinitesimal future directions in the tangent space to the manifold at that point; (b) there are no closed timelike loops.
- space-time is causally spatially isotropic: at any point of spacetime there is no preferred spacelike direction (this assumption does not imply spatial uniformity in the distribution of matter)
- space-time is causally temporally isotropic: there is no preferred timelike direction at any point of spacetime. For any two timelike directions at a given point of spacetime, there is a causal diffeomorphism of spacetime onto itself that maps one of these directions on the other

- spacetime can be globally factorized into $time \times space$

- spacetime is causally temporarily homogenous: translations with respect to the $time \times space$ factorization form up a group of causal automorphisms of spacetime, the temporal group belonging to this factorization; the energy is invariant under a group of $causal$ temporal translations related to a factorization of spacetime as $time \times space$

- spacetime is spatially homogenous

Segal showed [13] that these axioms are satisfied by only two possible manifolds: Minkowski $M_0 = \mathbb{R} \times \mathbb{R}^3$ and $M = \mathbb{R} \times \mathrm{S}^3$.

The group $\mathrm{SO}(2,4)$ is the 15-parameter conformal group of Minkowski space-time. (Sometimes its double cover $\mathrm{SU}(2,2)$ is chosen, which is locally isomorphic to $\mathrm{SO}(2,4)$.) The Lie Algebra $\mathrm{so}(2,4)$ is composed of 10 Poincare generators, $M_{\mu\nu}$ (space rotations and boosts) and P_μ (translations), together with scale transformation D and special conformal generators K_μ. By a theorem by Alexandrov and Zeeman [37], causality preserving transformations on Minkowski space are conformal transformations.

One can compactify the Minkowski space M_0, with $\mathrm{O}(1,3)$ as its symmetry group, by including it into the projective light cone (i.e. the space of all null lines through the origin) in a 6-dimensional Euclidean space $\mathbb{R}^2 \times \mathbb{R}^4$ with (2,4) signature [13]. The group $\mathrm{SO}(2,4)$ naturally acts on this space. Segal discussed compactification of Minkowski space in two ways: as a manifold with a $\mathrm{U}(2)$ group action; and as a projective quadric in 6-dimensional real space of signature $(2,4)$.

Darboux [35] introduced higher-dimensional *polyspherical* coordinates for higher-dimensional spaces with a point at ∞ included. Given $x^0, x^1, x^2, ...x^{n-1}, k, q \in \mathbb{R}$, with $x^0, x^1, x^2, ...x^{n-1}$ the Cartesian coordinates of a space $\mathbb{R}^n$ with a Euclidean or pseudo-Euclidean metric form, one can define *polyspherical* coordinates $y^0, y^1, ...y^{n+1}$ by

$$\begin{cases} y^\mu = x^\mu k, \quad \mu = 0, ...n-1 \\ y^n = (k+q)/2, \quad y^{(n+1)} = (k-q)/2 \end{cases} \tag{1}$$

If the metric is Lorentzian,

$$(x,x) = (x^0)^2 - (x^1)^2 - (x^2)^2 - ... - (x^{n-1})^2, \tag{2}$$

then the associated bilinear quadratic form in *polyspherical* coordinates is

$$Q(y,y) = (y^0)^2 + (y^{n+1})^2 - (y^1)^2 - (y^2)^2 - ... - (y^n)^2, \quad k = y^n + y^{n+1}, \quad q = y^n - y^{n+1}. \tag{3}$$

The light-cone at any point of Minkowski space is defined by a set of null-vectors $g(x, x) = 0$, with g a metric form. Similarily, a quadric Q in $\mathbb{R}^6$ with the $\mathrm{SO}(2, 4)$ action defines a natural Lorentz structure, which remain invariant under the action of $\mathrm{SO}(2, 4)$ group. The group of causal conformal transformations on $\mathbb{R}^6$ are those for which $Q(y, y) = 0$. One can rewrite

$$Q(y, y) = -(y^0)^2 - (y^5)^2 + (y^1)^2 + (y^2)^2 + (y^3)^2 + (y^4)^2, \tag{4}$$

which shows that the conformal group is related to the group $\mathrm{O}(2, 4)$. Because the coordinate sets $(y^0, y^1, y^2, y^3, k, q) \simeq \lambda(y^0, y^1, y^2, y^3, k, q)$ are equivalent up to a multiplicative factor λ with points in Minkowski space M_0, one can rewrite $Q(y, y) = 0$ as

$$(y^0)^2 + (y^5)^2 = 1 = (y^1)^2 + (y^2)^2 + (y^3)^2 + (y^4)^2. \tag{5}$$

Thus the compactified Minkowski space on which the conformal group acts continuously is compact and topologically isomorphic to $\overline{M} \simeq (\mathrm{S}^1 \times \mathrm{S}^3)/\mathbb{Z}_2$, or that time is compactified to S^1 and space to S^3. Time becomes periodic. To avoid this, one has to use the universal covering space $M \simeq \mathbb{R} \times \mathrm{S}^3$ and find the correspondence between six *polyspherical* coordinates on $\mathbb{R}^6$ and four coordinates in Minkowski space M_0.

The manifold $\widetilde{M} = \mathrm{S}^1 \times \mathrm{S}^3$ is the two-fold covering space of $\overline{M}$, while $M = \mathbb{R} \times \mathrm{S}^3$ covers it infinite number of times. The Minkowski space M_0 can thus be regarded as an open dense submanifold of $\overline{M}$ which is covered infinitely many times by M. The space $\mathrm{S}^1 \times \mathrm{S}^3$ admits a local notion of causality, but it not causal globally. The space $\mathbb{R} \times \mathrm{S}^3$ is the universal covering space of the conformal compactification $\overline{M}$ of Minkowski space M_0 which is globally causal [13].

Minkowski space M_0 can also be compactified as the group manifold of the unitary group $\mathrm{U}(2)$ via the Cayley transform. Minkowski spacetime can be represented by the causally isomorphic real linear space $H(2)$ of 2×2 Hermitian matrices. For a point $P \in M_0$ with coordinates $(x^0 := ct, x^1, x^2, x^3)$, with c the speed of light, the corresponding matrix A is:

$$\mathrm{A} = \begin{pmatrix} x^0 + x^3 & x^1 + \mathrm{i}x^2 \\ x^1 - \mathrm{i}x^2 & x^0 - x^3 \end{pmatrix} \in H(2)$$

This defined a map $\alpha : M_0 \to H(2)$. The vector space $H(2)$ of 2×2 Hermitian matrices can be causally immersed as a dense subset of the compact group $\mathrm{U}(2)$ of 2×2 unitary matrices as follows. For a Hermitian matrix A, the Cayley transform $\mathrm{U(A)}$ is the corresponding matrix:

$$\mathrm{U(A)} = (\mathbb{I} + \tfrac{1}{2}\mathrm{iA})(\mathbb{I} - \tfrac{1}{2}\mathrm{iA})^{-1} \tag{6}$$

where $\mathbb{I}$ is the identity matrix. The Cayley transform, $\beta : \mathrm{A} \mapsto \mathrm{U}(2)$, is one-to-one and, importantly, causal. It has a unique inverse, which is the generalized stereographic projection,

$$\mathrm{A} = -2\mathrm{i}(\mathrm{U} - \mathbb{I})(\mathrm{U} + \mathbb{I})^{-1}, \tag{7}$$

well defined as long as $\det(\mathrm{U} + \mathbb{I}) \neq 0$. The generalized stereographic projection is an analogue of the mapping between the unit circle in the complex plane, a multiplicative Lie group, and the imaginary axis its Lie algebra. The conformal infinity is the subset of $\mathrm{U}(2)$ consists of those matrices $\mathrm{U} \in \mathrm{U}(2)$ for which $\det(\mathrm{U} + \mathbb{I}) = 0$. The group $\mathrm{SU}(2,2)$ acts naturally on $\mathrm{U}(2)$.

The compactification $\mathrm{U}(2)$ of $H(2)$ can be lifted to its universal covering space $M = \mathbb{R} \times \mathrm{S}^3$. The group $\mathrm{SU}(2)$ is isomorphic to unit quaternions and is thus diffeomorphic to S^3, and $\mathrm{U}(2) \simeq \mathrm{U}(1) \times \mathrm{SU}(2)$. More precisely, the quotient $\mathrm{U}(2)/\mathrm{SU}(2)$ is isomorphic to $\mathrm{U}(1)$. The group $\mathrm{SU}(2)$ is diffeomorphic to S^3, thus $\mathrm{U}(2) \simeq \mathrm{S}^1 \times \mathrm{S}^3$. Since Minkowski spacetime is isomorphic to $H(2)$, it follows that $\mathbb{R} \times \mathrm{S}^3$ is the covering space of the compactification of Minkowski spacetime, $\mathbb{R} \times \mathbb{R}^3$. The following sequence of mappings [36] causally immerses Minkowski spacetime M_0 into the Segal-Einstein universe M:

$$M_0 = \mathbb{R} \times \mathbb{R}^3 \xrightarrow{\alpha} H(2) \xrightarrow{\beta} \mathrm{U}(2) \xrightarrow{\sim} \mathbb{R} \times \mathrm{SU}(2) = \mathbb{R} \times \mathbb{S}^3 = M. \tag{8}$$

SEGAL'S COSMOLOGICAL REDSHIFT

Minkowski space M_0 can be thought of as the tangent space at any point of $\mathbb{R} \times \mathrm{S}^3$, just as the complex plane is tangent to the Riemann sphere. However, the immersion of M_0 into $\mathbb{R} \times \mathrm{S}^3$ does not preserve the time coordinate or the space coordinates in the factorizations of these space-times as "space $\times$ time".

Local observations of dynamical quantities are represented not necessarily by generators of true, global symmetries, but by generators of corresponding symmetries in the flat tangential space. While angular momenta remain unchanged, the energy and linear momenta differ. The true energy is no longer represented by $-\mathrm{i}\hbar\frac{\partial}{\partial t}$, but by an operator $-\mathrm{i}\hbar\frac{\partial}{\partial \tau}$ where τ is the global time. Time and energy differ crucially in the two models [13] .

Assuming that the global, physical time τ is that derived from the $\mathbb{R} \times \mathrm{S}^3$ factorization, and that Minkowski time t is only a local projection of it, the redshift of photons propagated over large distances is obtained from the conformal invariance of Maxwell's equations and the requirement that the action of the time evolution groups, both standard (Minkowski) and non-standard ($\mathbb{R} \times \mathrm{S}^3$), is unitary. The global time τ and Minkowskis time t coordinates are related by the equation [13]

$$t = \frac{2R}{c} \tan\left(\frac{c\tau}{2R}\right) \tag{9}$$

where R is the radius of a 4-dimensional ball whose boundary S^3 constitutes our 3-dimensional space, and c is the speed of light. The two times correspond to two concepts of energy. The global, cosmic energy in $\mathbb{R}\times\mathrm{S}^3$ is conserved, while the photon energy measured in Minkowski $\mathbb{R}\times\mathbb{R}^3$ is reduced by the redshift. The redshift-distance relation in Segal's model [13] constitutes a verifiable prediction of the dependence of this redshift z on the geodesic distance $l = c\tau$ on S^3

$$z = \tan^2\left(\frac{c\tau}{2R}\right) = \tan^2\left(\frac{\rho}{2}\right) \tag{10}$$

The dimensionless quantity $\rho = l/R$, or a 4-dimensional analogue of the polar angle, runs from 0 to π when light is traveling from the emission point to its antipode in S^3. The redshift becomes infinite when the light goes around through a half-turn around the $\mathbb{R}\times\mathrm{S}^3$ universe to the observer. The redshift-distance relation can be also derived geometrically [28].

SEGAL'S COSMOLOGY, MATTER AND INTERACTIONS

In the $\mathbb{R}\times\mathrm{S}^3$ space-time, Maxwell's equations remain intact, as they are conformally invariant. The solutions to Maxwell's equations in Minkowski space extend uniquely to their solutions in the $\mathbb{R}\times\mathrm{S}^3$ universe [23]. The same holds for the Dirac equation, and for the Yang-Mills equations, which describe fermions and the strong and weak interactions in particle physics. These equation are conformally invariant in absence of matter, which allows to relate the Dirac and Young-Mills theories on Minkowski space-time with their analogues on a manifold with the boundary.

Special relativity is a limiting case of Segal's conformal theory, as the radius R becomes infinity. Locally, the two geometries are indistinguishable. Einstein's general relativity relates gravitation to curvature of space. In Einstein's original equations, flat Minkowski space is the simplest solution to the vacuum field equations of an empty universe. Einstein's modified equations include a cosmological constant term introduced to allow for a non-expanding universe in the presence of matter. However, the modified equations are the most general equations satisfying the usual minimal conditions. They allows for an empty space to have curvature, as in Segal's model. The inclusion of this term does not result in any inconsistencies with General Relativity. The relation of Segal's universe to general relativity is analogous to that for special relativity, except that the geometry of empty space is $\mathbb{R}\times\mathrm{S}^3$ rather than $\mathbb{R}\times\mathbb{R}^3$. In the limit of $R\to\infty$, the $M = \mathbb{R}\times\mathrm{S}^3$ universe becomes Minkowski $M_0 = \mathbb{R}\times\mathbb{R}^3$ spacetime. Segal's theory does not assume general relativity, but is compatible with it [13].

To quote Segal himself [19]: "How is general relativity and its relation to cosmology affected? The postulated infinitesimal structure of space-time in general relativity, i.e. of reference or empty space-time, is changed from a Minkowski space, formed from the tangent space at the point of observation, to a chronometric space, $\mathbb{R}\times\mathrm{S}^3$, invariantly attached to the point as the universal covering space of the conformal compactification of the tangent space with respect to the metric given in it. As far as is now known, the radius of the S^3 is too large (in conventional units; in natural units, the S^3 is of unit radius) to produce any presently observable effects in the small, and local observable aspects of general relativity are therefore unaffected. In the large, because of the compactness of S^3 it is necessary, as Einstein proposed, to add the cosmological term to his equation. Overall, the resulting universe departs widely from the Friedman-Lemâitre model—any expansion, if present at all, must be slight-but in its gross features is consistent with Einstein's original static conception."

PRELIMINARY COMPARISONS OF MODELS OF UNIVERSE WITH DATA

The predicted redshift-distance relations are different in Segal's model and in the expansionary SCM model [38,39]. At least in principle, the data itself can differentiate between the two models. Segal published several papers showing that the data available at his time agreed with the predictions of his $\mathbb{R} \times \mathrm{S}^3$ universe. However, since his passing in 1998, enormous progress has been made in observational astronomy. Modern galactic surveys are covering larger areas of the sky and and provide data from greater redshifts than have been probed previously. Here, we set out to investigate whether Segal's model can be falsified with modern data.

THE MAGNITUDE-REDSHIFT RELATION

For distant objects, distance is not a directly observable quantity, and it can only be estimated using various proxies. If one assumes that the objects are standard candles with the same absolute luminosity, the purely geometric relations between apparent luminosity and distance allows comparison of correlations between the observed magnitude and redshift, $m(z)$. The data for type 1a supernovae, the best known standard candles, agrees very well with the SCM, but it also agrees with Segal's model. In Figure 1, the data from the Supernova Cosmology Project [40] compilation Union2.1 is shown along with theoretical predictions from the SCM (red) and from Segal's $\mathbb{R} \times \mathrm{S}^3$ cosmology (green). More information about the theoretical predictions used, and the fits themselves can be found in Appendix 1.

One should keep in mind that the comparison, although in principle very simple, is not trivial due to possible but unknown effects of extinction of light from distant sources and details of star evolution in time.

THE NUMBER COUNT $N(< z)$ RELATION

Another independent observable is the number count, or the number of objects of a given type seen in a fixed cone versus their redshift, $N(< z)$. Assuming a uniform distribution of objects in the universe, $N(< z)$ is directly

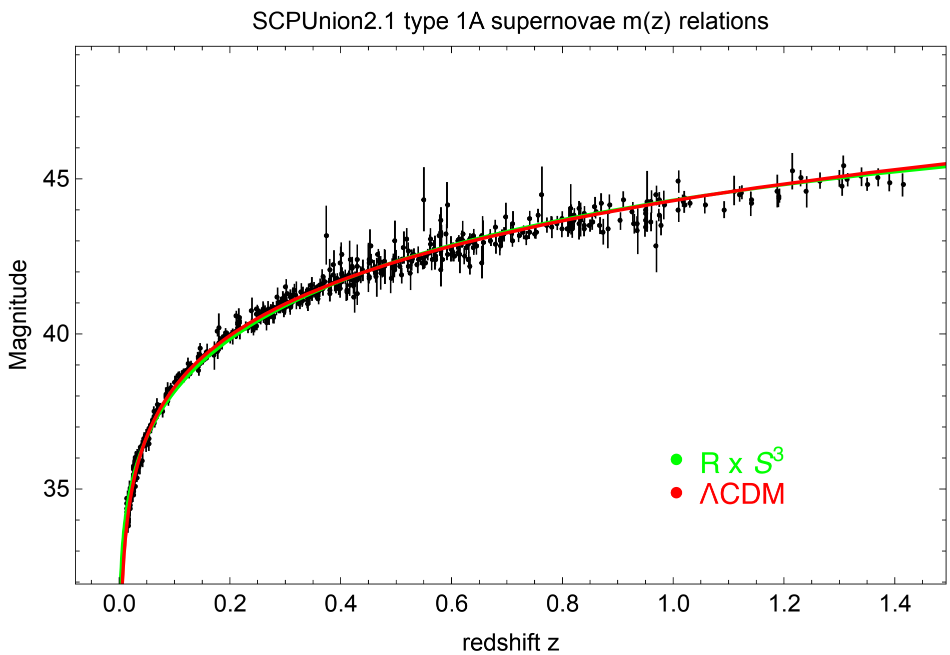

Figure 1: The SCP "Union2.1" SN Ia compilation is an update of the "Union2" compilation. Plotted are observed magnitudes as a function of redshift for 580 SNe type Ia that pass usability cuts. The red curve is the result of fits based on Standard Cosmological Model [38] of $m(z)$ within SCM with $q_o = 1/2$ for a flat space, and including a correction for possible light extinction. The green curve id based on Segal's model $\mathbb{R}\times\mathrm{S}^3$. The prediction from Segal's model includes a correction for the observed intensity to account for possible extinction, $I_{corr} = I\ \exp(-\lambda c\tau)$, corresponding to the extinction length of $1/\lambda \approx \frac{1}{2}R$, the radius of the universe. The two fits have the same number of free parameters. Fits to a newer parametrization [39], assuming zero curvature term $\Omega_k = 0$, and taking $\Omega_{matter} = 0.276$ for the matter term (red), as determined independently from the CMB studies, gives virtually identical results to those obtained with the SCM parametrization used in the fit [38].

proportional to the volume, $V(< z)$, enclosed in this cone, and is thus sensitive to the geometry of space. Plotting $N(< z)/N(< z_{max})$, where z_{max} is the maximum redshift in a sample, and comparing it to $V(< z)/V(< z_{max})$ as a function of redshift z can, in principle, differentiate between possible geometries of the universe.

The data from several Astrodeep Hubble Frontier Fields [41], based on a combination of observations from the Hubble Space Telescope, the Spitzer telescope, and the ground-based VLT Hawk-I is found to be in agreement with predictions of Segal's model. In Figure 2, we show results on $N(< z)$ as a function of the reshift z for the Astrodeep Abell 2744 field, and in Figure 3 we show the magnitude-redshift relation, $m(z)$, based on the same data, for completeness. We selected only those objects in the Astrodeep Abell 2744 field that have redshifts determined spectroscopically, or photometrically with the redshift uncertainty range $\mathrm{ZBEST\_SIQR} < 0.1$.

One should keep in mind that such comparisons may be affected by selection biases, by the unknown effects of extinction of light when it travels through distant parts of the universe, and by possible effects due to evolution of galaxies in time. For very large redshifts, there should be no galaxies in the SCM, as they need some minimum time to form after the Big Bang.

Interestingly, several recent papers based on data obtained with the James Webb Space Telescope, reported observations of distant galaxies of uncharacteristically large mass, given their high redshifts [42, 43]. According to the current ideas about evolution of galaxies in the expanding universe, such objects are not expected so early after the Big Bang. However, the presence of galaxies this large at such high redshifts is consistent with a static $\mathbb{R} \times \mathrm{S}^3$ universe, in which galaxies are distributed homogenously in the S^3 space, including distances corresponding to very large redshift values.

THE COSMIC MICROWAVE RADIATION

It is an important observational fact that our universe is filled with omnidirectional cosmic microwave background radiation (CMB) with the black body spectrum corresponding to temperature of approximately $T = 2.7\ K$. In the ΛCDM model, the CMB is explained as the light that was originally emitted from the surface of last scattering about 380,000 years after the Big Bang, now at redshift of z ~1100 [44].

In Segal's model the CMB corresponds to "residual light", light that has not been absorbed over multiple turns around the spatially closed $\mathbb{R} \times \mathrm{S}^3$ universe. Segal has shown that its energy distribution is expected to be the Planck black-body spectrum [23]. The only properties of light required in the proof are that it is described by Maxwell's equations, Bose-Einstein statistics, and that it is stochastically emitted and absorbed by matter via a temporally invariant interaction over many turns around the $M = \mathbb{R} \times \mathrm{S}^3$ universe.

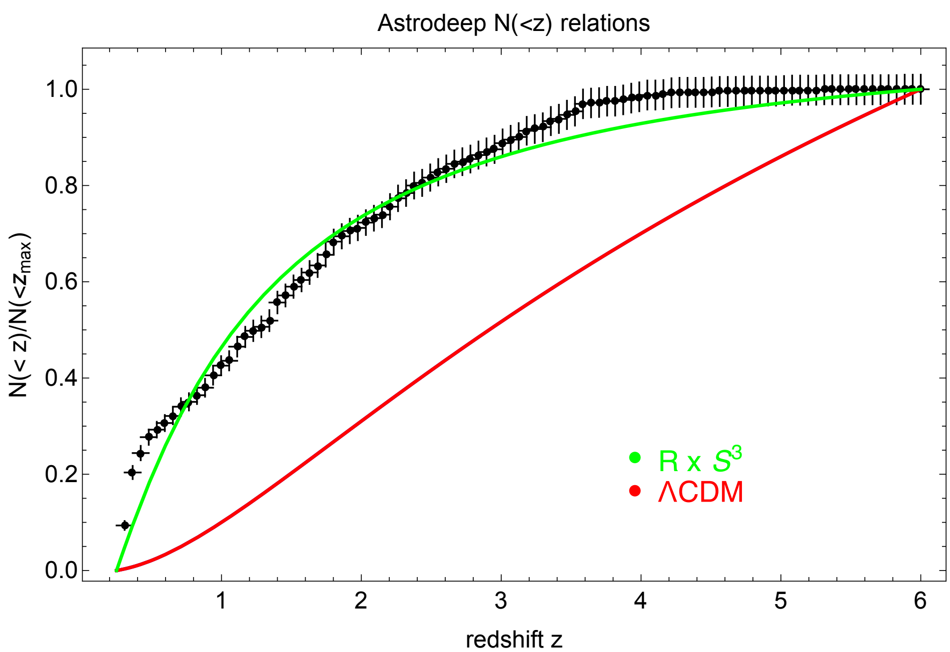

Figure 2: The normalized number count, $N(< z)/N(< z_{max})$ based on data from for one of Frontier Fields, Abell A2744, for objects that have their redshifts measured spectroscopically, or photometrically with the redshift uncertainty range ZBEST_SIQR < 0.1. The curves are the normalized volumes as a function of redshift z, $V(< z)/V(< z_{max})$ calculated for SCM (red) and Segal's $\mathbb{R} \times \mathrm{S}^3$ cosmology (green). The number count $N(< z)$ is proportional to the volume, $V(< z)$, enclosed in a chosen angular cone up to redshift z. The dependence of the volume contained within redshift z is $V(< z) \sim (1 - \frac{1}{\sqrt{1+z}})^3$ for SCM [38], and $V(< z) \sim \tan^{-1}\sqrt{z} - \frac{1}{4}\sin\left(4\tan^{-1}\sqrt{z}\right)$ for Segal's model [13], correspondingly.

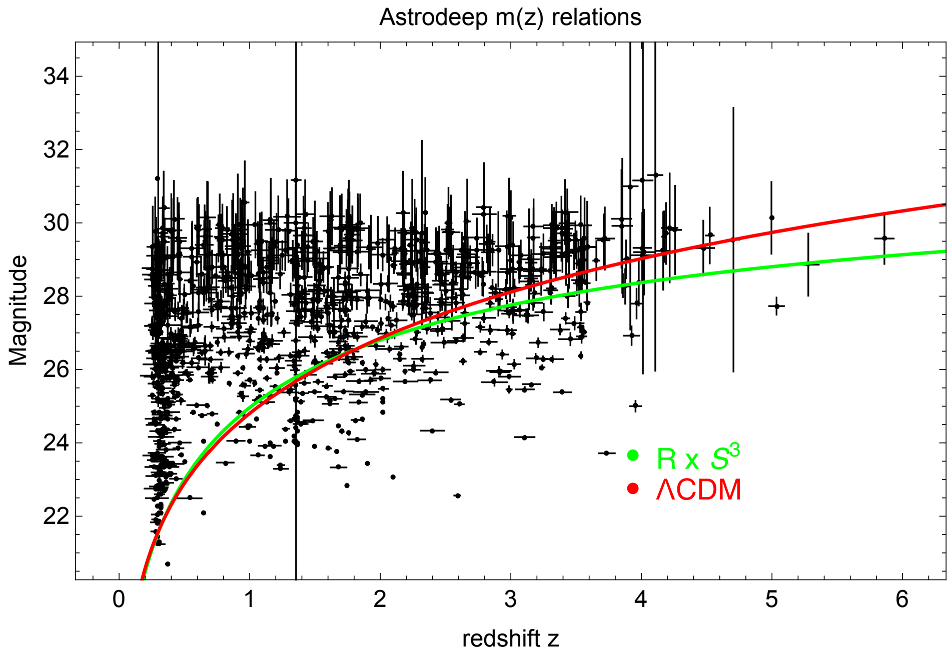

Figure 3: The observed magnitude as a function of redshift for objects from one of Frontier Fields, Abell A2744, for objects that have their redshifts measured spectroscopically, or photometrically with the redshift uncertainty range $ZBEST\_SIQR < 0.1$. The curves shown are results of fits based on Standard Cosmological Model [38] (red) and Segal's $\mathbb{R} \times \mathrm{S}^3$ (green), using the same parameters as obtained in fits to the SCP "Union2.1" supernovae data shown in Figure 1, except for an additive constant.

The absorption coefficient α per single half-turn around the universe is closely related to the fraction of the celestial sphere obscured by galaxies distributed uniformly in a closed S^3 space. In Segal's simplest model, light is absorbed by matter present in N galaxies represented by black disks of radius r. Since the number of galaxies in the compact S^3 is finite, $\alpha \ll 1$. The total energy flux of light that has not been absorbed over n half-turns is $P\mathrm{e}^{-\alpha n}$, where P is the energy flux of "pristine" light that did not yet travel by more than a half-turn around the universe. Summing a resulting power series over multiple half-turns around the universe gives

$$P_{\mathrm{CMB}} = \sum_{n=1}^{\infty} P\mathrm{e}^{-\alpha n} = P/\alpha \tag{11}$$

since $\alpha \ll 1$. P is the energy flux of "pristine" light, emitted by N galaxies distributed uniformly in the universe, averaged over the entire universe and taking redshift into account. We calculated the average energy flux P following a geometrical analysis analogous to an estimate of the absorption coefficient α. The "pristine" light originates as light emitted by N galaxies, represented by disks of radius r and all of the same typical luminosity. In this simple model, the number of galaxies N, their radii r, and the radius of the 4-D hypersphere R, cancel out. As a result, one can express P_{CMB} in terms of the energy flux of light emitted by a typical galaxy at some distance from Earth, in terms of the radius of a typical galaxy radius r times a numerical factor. Since the spectrum of residual light is the Planck's black-body distribution [23], one obtains a prediction for the temperature of the CMB from the Stefan-Boltzmann law.

The observed value of the CMB temperature $T = 2.7\ K$ can indeed be naturally explained. Taking the luminosity of Milky Way, $L = 5 \times 10^{36}\ W/m^2$ for the luminosity of a typical galaxy, our simple model gives $T = 2.74\ K$. (To give a sense of sensitivity of this prediction to assumed typical luminosity, if one takes the luminosity of Andromeda galaxy $(M31)$ as that of a typical galaxy, the model gives $T = 3.2\ K$, We note that Andromeda is a very bright galaxy, most likely more luminous than an average galaxy in the universe.)

It is also appears possible to explain the main features of the observed power spectrum of CMB fluctuations as due to the natural statistical distribution of the hierarchy of the large-scale structures in the universe - galaxy clusters, superclusters, voids et cetera.

One can show that the main feature in the CMB power spectrum, the first peak at $l \sim 200$, can be reproduced in this way, as shown In Figure 4. We have used a simple model in which galaxies are distributed according to a hierarchical sequence of distances between superclusters, clusters and galaxies, with the distance scales taken from existing observations, while keeping the number of galaxies per unit volume on large distance scales constant. We generated a network of galaxies' positions in the sky, together with their corresponding energy fluxes, taking into account reduction of the flux according to galaxies' distances from the observer, and their geometrical redshift. The shape of the power spectrum is sensitive to the assumed distance scales between superclusters, clusters, galaxies, and R, the "radius of the universe". We used software provided in the healpy package [45] to convert the sky coordinates to obtain maps of the sky using a chosen HEALPix pixelization scheme, and then calculated the power spectra of the simulated fluctuations. We used the pixelization scheme that was used for WMAP data, which did not have very high angular resolution. We believe this choice was appropriate to explore whether the first peak of the power spectrum can be explained by our very simple, preliminary model. Higher resolution data is available for future efforts to reproduce higher order characteristics of power spectrum. More information about the theoretical predictions used, and the fits themselves can be found in Appendix 2.

CONCLUSION

Segal's "Chronometric Cosmology", in which the geometry of the universe is spatially closed, finite and eternal, provides an alternative explanation for cosmological redshift, and provides a verifiable prediction on the redshift-distance dependence. We have compared the predictions of the Standard Model of Cosmology and Segal's universe

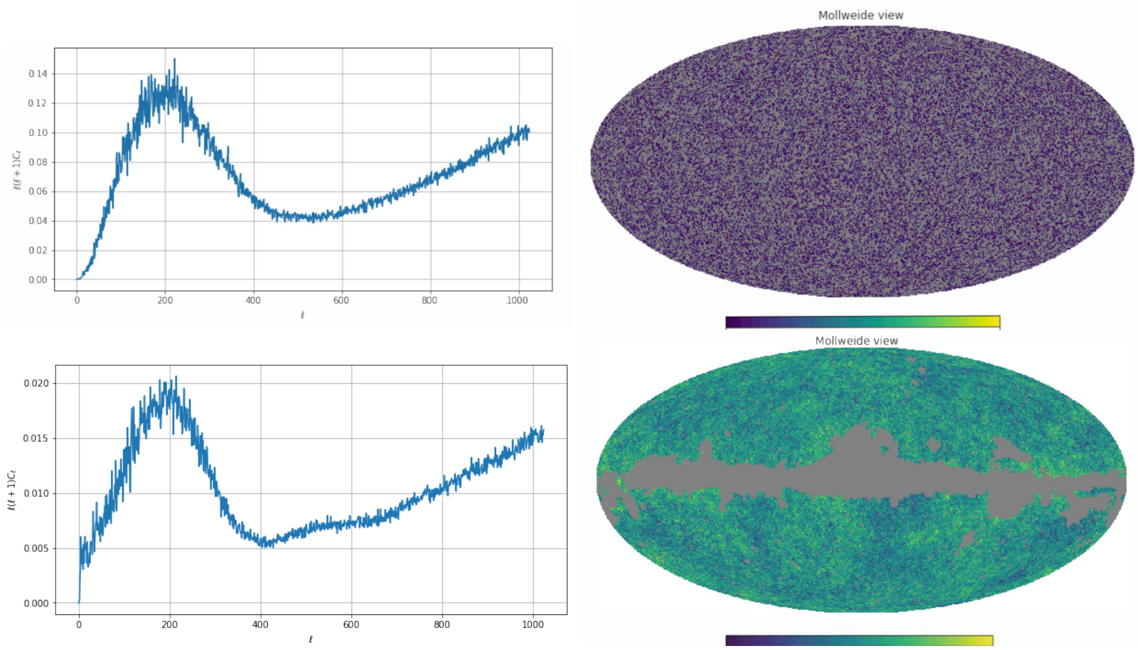

Figure 4: The simulated CMB power spectrum in Einstein-Segal universe (top left) calculated with healpy directly from a sky map of temperature fluctuations generated in Mathematica (top right) assuming a hierarchical distribution of galaxy superclusters, galaxy clusters and voids. For comparison, the corresponding CMB power spectrum calculated directly from WMAP is shown (bottom left) together with a WMAP data showing CMB temperature in our universe.

cosmology with data on cosmological redshifts, specifically $m(z)$ and $N(<z)$, and with the temperature and power spectrum of the CMB. Surprisingly, the data is consistent with predictions of Segal's $M = \mathbb{R} \times \mathrm{S}^3$ universe.

We believe further research is merited into Segal's alternative explanation of the cosmological redshift and his Chronometric Cosmology model. Following a more detailed investigation of the consistency of observational data with alternative models of the universe, future work should explore the questions of conservation of energy, creation or recycling of matter and whether the distribution of elements observed in our universe could be explained in a spatially closed, static and eternal Segal-Einstein universe.

Additionally, the next few years will bring more data from the already operational James Webb Space Telescope (JWST), which will extend the reach and the resolution of studies of distant galaxies and objects at high redshifts. A dedicated study of the $N(<z)$ relation for chosen fixed angular cones in one of the Deep Fields could provide important information about the geometry of the universe. This observable is directly sensitive to the geometry of space, as shown in Figure 2. The JWST data could also provide important information about extinction and absorption of light emitted by distant sources, and the evolution of stars and galaxies in time, which at present introduce complications to the interpretation of both $m(z)$ and $N(<z)$ studies.

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Appendix 1: Fits to $m(z)$ relations

In Figure 1, plotted are observed magnitudes as a function of redshift for the SCP "Union2.1" SN Ia compilation data. The SCM prediction for $m(z)$, has been evaluated with the expression derived by Sandage [38], equation 33, page 577:

$$m_{\rm bol}(z) = M_0 + 5\log_{10}(\frac{1}{q_o^2}(zq_o + (q_o - 1)(-1 + \sqrt{2q_o z + 1}))) + C \tag{12}$$

where $C = 2.5\log_{10}(4\pi) + 5\log_{10}(\frac{c}{H_o})$, with speed of light c and Hubble constant H_o. The parameter q_o was set to the value of $\frac{1}{2}$ for a flat universe and $M_{\rm bol}$, the absolute brightness, was included with C as a free parameter. In addition, we included in the fit a correction for the observed intensity to account for possible extinction, $I_{\rm corr} = I{\rm e}^{-\lambda c\tau}$, which gives an additional term $2.5\lambda\log_{10}(1+z)$ in the expression for $m_{\rm bol}(z)$, where λ is the extinction coefficient, a free parameter.

The prediction of Segal's for $m(z)$ has been obtained using the expression given by Segal [13] on page 94:

$$m(z) = 2.5\log_{10}z - 2.5(2-\alpha)\log_{10}(1+z) + C \tag{13}$$

where α is the power in the spectral function $f(\nu) \propto 1/\nu^\alpha$, where ν is the frequency. The parameter α was set to 1. Setting α to some other value, say 2-3, gives almost identical results, except for a different value for the parameter λ, which is adjusted by the fit while leaving the fit quality almost identical. We also included in the fit a correction for the observed intensity to account for possible extinction, which results in an additional term $5\lambda\log_{10}({\rm e})\tan^{-1}(z^{1/2})$ in the expression for $m(z)$. The two fits have the same number of free parameters. Both fits are good, the values of $\chi^2/{\rm dof}$, reported by Mathematica are 0.98 for SCM and 1.67 for Segal's model, where ${\rm dof}$—the number of degrees of freedom in the fit.

In Figure 3, plotted are the observed magnitude as a function of redshift for objects from one of Frontier Fields, Abell A2744. The curves shown in this figure are based on results of fits obtained with Standard Cosmological Model [38] (red) and Segal's $\mathbb{R} \times {\rm S}^3$ (green) and extrapolated to a wider redhift range. We have used the same parameters as obtained in fits to the SCP "Union2.1" supernovae data shown in Figure 1, except for an additive constant. Both curves describe the data well, however results are inconclusive. The values of $\chi^2/{\rm dof}$ reported by Mathematica are 2125.71 for SCM and 2092.68 for Segal's model.

Appendix 2: CMB temperature and the first peak in the CMB power spectrum

In Segal's model the CMB corresponds to "residual light", light that has not been absorbed over many turns around the universe. Following Segal's original approach, the absorption coefficient α is the fraction of the sky blocked by the randomly distributed N galaxies in S^3, each with radius r. The 4-dimensional analogue of the polar angle in S^3, at which all N galaxies should be placed to result in the same absorption coefficient as if they were uniformly distributed in S^3 space, is $\rho_{\text{eff}} = \pi/4$, or $\rho = 3\pi/4$. Since $\rho = l/R$, and $a = R\text{sin}\rho$, it follows that the effective radius a_{eff}, the radius of a slice of S^3 at which one may place all N galaxies to give the same absorption, is

$$a_{\text{eff}} = R\text{sin}(\pi/4) = R\text{sin}(3\pi/4) = R/\sqrt{2}. \tag{14}$$

The absorption coefficient α can be thus estimated by finding the ratio of areas obscured by N galaxies to the area of a slice of S^3 of the radius a_{eff}, or the area of a slice of S^3 at effective 4D polar angle ρ_{eff}:

$$\alpha = N\pi r^2/(\pi R^2(1 - \cos^2\rho_{\text{eff}} + \sin^2\rho_{\text{eff}}) \tag{15}$$

$$\alpha = N\pi\epsilon^2/(\pi(1 - \cos^2(\pi/4) + \sin^2(\pi/4)) = N\pi\epsilon^2/(\pi(1 - \cos^2(\pi/4) + \sin^2(\pi/4)) \tag{16}$$

which gives $\alpha = \epsilon^2 N/2$, where $\epsilon = r/R$.

As described in the main text, $\alpha \ll 1$ and the total energy flux of light that has not been absorbed over n half-turns is $Pe^{-\alpha n}$, where P is the energy flux of "pristine" light. Summing a resulting power series gives

$$P_{\text{CMB}} = \sum_{n=1}^{\infty} P\text{e}^{-\alpha n} = P/\alpha \tag{17}$$

To calculate the average flux of pristine light at a typical point of the S^3 universe, or the energy flux Φ emitted by N galaxies of luminosity L, randomly distributed in S^3 universe, we used the effective distance $a_{\text{eff}} = Rsin\rho_{\text{eff}}$ at which all N galaxies should be placed to give the same flux $\rho = \pi/4$ or $\rho = 3\pi/4$.

$$\Phi = NL/(4\pi a^2) = NL/(2\pi R^2(1 - \cos(\pi/4)^2 + \sin(\pi/4)^2)) = NL/(2\pi R^2) \tag{18}$$

P is the energy flux of "pristine" light, emitted by N galaxies distributed uniformly in the universe, averaged over the entire universe and taking redshift into account. The observed energy flux of the pristine light will be reduced by the redshift z. In Segal's model, after integration over the entire space, $\Phi_{\text{corr}} = P/2$. The luminosity of a typical galaxy L can be estimated from the energy flux of pristine light observed on Earth from Milky Way Φ_E, $L = \phi_E 4\pi(r/2)^2$, where we took $r/2$ for the distance from Earth to Milky Way center. The expected flux of CMB is thus the product of corrected energy flux of pristine light at a typical point of the universe and the enhancement factor $1/\alpha$,

$$P_{\rm CMB} = \Phi_{\rm corr}/\alpha = NL/(2\pi R^2)/(\epsilon^2 N/2)/2 = N(\Phi_E 4\pi(r/2)^2)/(2\pi R^2)/(\epsilon^2 N/2)/2 = \Phi_E/2 \qquad (19)$$

Alternatively, one can take various estimates of luminosity of Milky Way, or Andromeda, and use their distance from Earth to find the flux observed on Earth. For Andromeda, the expressions will be modified by the ratio of its distance from Earth to the distance from Earth to the center of Milky Way. We obtained the temperature of a black body corresponding to the predicted energy flux $P_{\rm CMB}$ using Stefan-Boltzmann law: $T = (P_{\rm CMB}/\sigma)^{1/4}$, where σ is the Stefan-Boltzmann constant.

The power spectrum for CMB characterizes two-point correlations between fluctuations of CMB temperature measured in pixels according to a scheme defined in the healpy package. For the WMAP data, which detector had limited angular resolutions, the number of pixels was smaller than for Planck data. We used the same pixelization scheme to compare with WMAP, to allow a comparison of main features of the power spectrum with our crude model. In our approach, we are adding up the energy flux from all sources, assumed to be galaxies of some typical luminosity, falling into a given pixel. Since we are integrating over the entire volume of $\rm S^3$ space, and account for the dependence of the flux on the distance from the observer, this method provides an estimate of the average energy flux of "pristine light in the universe. This flux is related to the energy flux of CMB. In Segal's model, CMB is the residual light, not absorbed over many turns. It is larger than the average flux of pristine light by a factor $1/\alpha$, where $\alpha \ll 1$ is the absorption coefficient, as described in the main text. However, the flux of pristine light a typical point of the universe is smaller than the flux of pristine light on Earth, as Earth is located very close to the center of Milky Way. This dilution factor is calculated similarly to the enhancement factor. As a result, since many factors cancel out, the CMB flux in Segal's model is expected to be the product of some numerical factor f and the energy flux from a typical galaxy at a chosen distance from an observer.

To generate the power spectrum we used a crude simulation with the number of superclusters $N_{\rm supercl} \sim (0.1 - 0.5) \times 10^6$, the number of clusters in a supercluster $N_{\rm cl} = 7 - 10$, the number of galaxies in a cluster $N_{\rm g} \sim 20 - 50$, and the "radius of the universe" $R > 200 - 300$ Mps. The distance between superclusters was assumed to be $D \sim 10$ Mps, and this value was taken as the "thickness" of $\rm S^3$ slices, surfaces of spheres S^2 of varying radii a. The distances between galaxy clusters and galaxies were obtained by scaling D by $1/N_{\rm cl}$ and $1/N_{\rm g}$, correspondingly. The number of superclusters in each slice was taken such that the density of superclusters per unit volume was kept approximately constant. In each slice of $\rm S^3$, we used Mathematica [46] to generate the supercluster positions on the spheres of radii corresponding to a given slice of $\rm S^3$. The cluster positions were then distributed randomly on a spherical surface surrounding the generated supercluster positions, using a typical distance between clusters as the appropriately smaller radius. Finally, galaxies were randomly generated around he position of clusters to which the galaxies belonged. Each generated galaxy has been assigned three coordinates (ρ, θ, ϕ), which allows to account for geometric effects and to calculate the energy flux reaching the observer, including those of the redshift. To generate a projection on the "celestial sphere" we kept the two angular coordinates, (θ, ϕ), after converting them to the galactic coordinates.

Segal proved that the energy spectrum of light circulating multiple times around the universe will be a black-body spectrum [23]. The total energy flux of the simulated pristine light falling into a given pixel, multiplied by the factor f, gives the expected energy flux of CMB in a given pixel. Taking the fourth root of the simulated CMB energy flux gives the temperature T of a black body that would give the same energy flux. In this way we obtained a simulated pixelized map of CMB temperature. Finally, we used the healpy package to obtain the power spectrum of the fluctuations of pixel temperatures.

The range of values that seem to reproduce the first peak in the CMB power spectrum at $l \sim 200$ is quite wide, it is the ratios of the distance scales that are important. However, the sensitivity of the position of the first peak in the CMB power spectrum to R opens a possibility, in principle, to estimate the range of values of R allowed by the CMB data.

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